AN OBJECTIVE COMPARISON BETWEEN CUMULANT-BASED AND BURG'S PARAMETRIC MODELING OF WEIBULL RADAR CLUTTER IN THE SPECTRAL AND ENSEMBLE DOMAINS: A CASE STUDY

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ABSTRACT

A case study of a Weibull distributed radar sea clutter modeling is the way chosen to state an objective numerical correspondence between the spectral and ensemble domains. Once the mapping is stated, it is used to compare conventional second-order statistics AR modeling with third-order cumulant-based AR modeling, leading to significant decrease in model order.

I. INTRODUCTION

In present radar technology, the problem of clutter modeling for purposes of classification or suppression has been the main subject of research for many radar engineers. Clutter may be defined as any undesired echo in the radar. Looking at the literature, we can identify three main guidelines, often merged, that authors adopt to deal with the problem: processing in the ensemble, spectral and time domain.

When an attempt is made to statistically characterize the clutter process through moments and probability density functions, we say that the related data is being processed in the ensemble domain. The analysis is said to be in the spectral domain when the primary interest is to find out the power spectral density and related parameters. Once spectrum estimation techniques often require the stochastic process to satisfy a restricted set of conditions to be applicable, processing in the ensemble domain is frequently required as a preceding step. These restrictions, that include gaussianity and stationarity for the majority of existing autocorrelation-based spectral estimators, are needed in order to guarantee statistical and numerical consistency of the results. At last, the time domain processing of clutter deals with some type of real-time processor, whose design is based on statistical or spectral knowledge of the process.

Many signal processing and system theory problems involve significantly non-Gaussian signals exciting non-minimum phase systems. Even a Gaussian signal passing through a nonlinear propagation media (or filtering operation) becomes non-Gaussian at the output. It is well known that radar returns are modeled as Bernoulli-Gaussian, generalized Gaussian, Rayleigh, log-normal, or Weibull distributed processes. In this case the standard energy analysis based on second order moments and power spectra fails to provide a complete statistical description of these non-Gaussian signals, and is unable to convey complete phase information about the underlying non-minimum phase model. The autocorrelation provides a complete statistical description only in the case of Gaussian linear processes. Therefore, cumulants (in the spectral domain called polyspectra) have received attention of the statistics, signal processing and system theory literature, for analyzing non-Gaussian linear, or Gaussian nonlinear processes. Recently, parametric MA, AR and ARMA models have been proposed to fit the cumulants of non-Gaussian processes [2], [3], [4], [14].

In this work, the cumulant approach objectives are spectral modeling, model order reduction and decrease of observation additive noise influence. Up to now, research on clutter modeling has concentrated mostly on exploiting second-order information of the output random process, yielding to equivalent parametric minimum phase models, more suitable for spectral estimation rather than stochastic realization and identification of realistic systems. In this paper, a case study of radar sea clutter AR modeling is the way chosen to carry out a discussion about the new results obtained, when a cumulant-based approach is adopted. Attempting to the non-Gaussian characteristic of the process, a comparative study is presented, that involves the already available results in the ensemble domain [12], the conventional results from the power spectrum domain (energy-based approach via Burg's algorithm) [5], [7], [8], [10], and the results in the bispectrum domain, where third-order cumulants are used [6]. These new results include a reduction in the AR order required, a more realistic associated autocorrelation sequence, and a less biased corresponding power spectrum, all these characteristics achieved due to the reduced receiver

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noise influence over the higher-order statistics, that must be extracted from data.

The organization of this paper is as follows: the needed background is briefly introduced in section II. Section III states the problem of sea clutter modeling and brings the description of the specific case under study, including the ensemble domain results obtained by Sekine in a previous paper [12]. Section IV compares these results to that ones obtained from a conventional second-order moment based AR modeling (spectral domain), using the most appropriate non-adaptive algorithm for short sequences (locally stationary processes): the Burg's algorithm. Still in section IV, a comparison is performed between the AR model obtained via third-order cumulants. Conclusions are drawn in section V.

II. BACKGROUND

The Ensemble Domain

The *moment generating function* M(t) is defined for a random variable (r.v.) x as [11]:

$$M(t) = \mathbf{E}\{e^{tx}\} \tag{1}$$

Considering that M(t) exists for $|t| \le T$, and that μ_r is the *rth* moment around origin, we can expand M(t) as power series:

$$M(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mathbf{E}\{x^r\} = \sum_{r=0}^{\infty} \mu_r \cdot \frac{t^r}{r!} \tag{2}$$

The moments are not guaranteed to exists for all distributions. In these cases, M(t) won't exist in any interval that contains the origin. A modified version of this function, guaranteed to exist for any distribution [11] is the *characteristic function* Φ(u). Consider a complex value t, so as M(t) is now defined in the complex plane. If any of the moments does not exist in the real axis, it will always exist in the imaginary axis. Making t=ju (u real), Φ(u) is defined as:

$$\Phi(u) = M(ju) = \mathbf{E}\{e^{jux}\} \tag{3}$$

We can now state the *cumulant generating function* for r.v. x, also known as the *second characteristic function* K(t) as:

$$K(t) = \ln\{\, M(t)\} = k_1 t + k_2\frac{t^2}{2!} + \cdots + k_r\frac{t^r}{r!} \tag{4}$$

where the k_r coefficients are the *rth* cumulants of the r.v. x, which are strictly related to the moments, and t=ju. For the four first cumulants, the following relations apply:

$$k_1=\mu_1=\bar{x}\,, \quad k_2=\mu_2=\sigma^2, \quad k_3=\mu_3\,, \quad k_4=\mu_4 - 3\sigma^4 \tag{5}$$

Generalizing the above concepts for a collection of r.v. forming an stochastic process (s.p.) over time, the cumulant's characteristic of primary interest here is that it provides a measure of the "distance" from a Gaussian s.p.: the third-order cumulant measures the skewness of its PDF in respect to a Gaussian equivalent PDF, and the fourth-order cumulant measures its smoothness. Let $M_N(t)$ and $K_N(t)$ be the respective generating functions for a Gaussian s.p. of x:

$$M_N(t) = e^{(\bar{x}\cdot t + \frac{1}{2}\sigma^2 t^2)} \tag{6}$$

$$K_N(t) = \ln\{M_N(t)\} = \bar{x}\cdot t + \frac{1}{2}\cdot\sigma^2 t^2 \tag{7}$$

So, we conclude that for the Gaussian distribution:

$$k_r = 0, \quad r \ge 3 \tag{8}$$

This result is specially important when we deal with non-Gaussian s.p. contaminated by Gaussian noise. In this context, we can say that cumulant's stochastic description for orders greater than two are not subject to Gaussian noise contamination. Some symmetries can be extracted from cumulants of weakly stationary s.p., making its calculations manageable. Cumulant estimators are also stated in the references [2], [4], [6]. As it is not the intent here to discuss the details of the algorithms adopted, let's proceed in another direction.

The Spectral Domain

Suppose we have a time series y[k] that can be identified as a wide-sense stationary (WSS) stochastic process with power spectral density (PSD) $P_{yy}(z)$, and a linear time-invariant (LTI) system absolutely stable with rational transfer function H(z). If we apply white-noise to the system input, we have the *spectral factorization theorem* [8] which states that a rational H exists so that:

$$P_{yy}(z) = H(z)\cdot H^*(1/z^*) \tag{9}$$

where the poles and zeros of H are inside the unit circle. So, we can look at all WSS s.p. as "the output of a dynamic LTI system excited by white noise", which is also known as the *representation theorem*. The parametric modeling problem for spectral estimation consists in determining, from data, the parameters of the whitening filter H(z), so that it incorporates the maximum possible correlation embedded in the time series, ideally leading to a white-noise-like residue sequence. This problem forms the base of Wiener-Hopf theory.

Similarly, for cumulant-based spectral estimation, we make use of the following theorem (*Brillinger & Rosenblatt - 1967*, In:[2]): consider H(z) is the transfer function of a causal, exponentially stable, LTI system. Consider its input v[n] as an independent and identically distributed (i.i.d.) sequence, with the *rth*-order cumulant k_r given as:

$$k_r^v(0, m_1, \cdots, m_{r-1}) = \begin{cases} \gamma_r^v & \text{, if } m_1 = \cdots = m_{r-1} = 0 \\ 0 & \text{, for other cases} \end{cases} \tag{10}$$

where γ_r^v is the *rth*-order cumulant of the random variable v, which exists if the *rth*-order and less moments exist. So, the output polyspectrum of order r-1 is given for the output time series y[n] as:

$$P_{r-1}^{yy}(z_1,\cdots,z_{r-1}) = \gamma_r^v H(z_1)\cdots H(z_{r-1})H(-[z_1\cdots z_{r-1}]^{-1}) \quad (11$$

where $z_i = e^{j\omega_i}$. For r=3, P_2^{yy} is called bispectrum.

The following theorem provides a means of using polyspectra in the stochastic realization of non-minimum phase LTI systems (*Brillinger & Rosenblatt - 1982,* In:[2]): if the input is non-Gaussian, i.i.d., with finite moments, and $H(1) \neq 0$, both amplitude and phase of the system H(z) can be retrieved (unless for sign and linear phase ambiguities) from one of the output polyspectra of order r-1, where r>2. Once for a Gaussian s.p. g[n], $\gamma_r^g = 0$ for all r>2, the input process must be non-Gaussian.

With this statements in mind, the main results involving higher-order statistics in s.p. parametric modeling and spectrum estimation can be summarized as:

(1) If the process is Gaussian and H(z) is minimum-phase, autocorrelation-based (2nd-order statistics) methods will identify correctly both magnitude and phase.

(2) If v[n] is Gaussian and H(z) is non-minimum phase, no procedure can recover the real phase of the system.

(3) If the process is non-Gaussian and H(z) is non-minimum phase, autocorrelation-based methods will correctly identify the magnitude of H(z), but not its phase.

(4) If the process is non-Gaussian and H(z) is non-minimum phase, the true magnitude and phase can be obtained from the *knowledge* of v[n]'s non-Gaussian distribution. This requires the solution of non-linear equations.

(5) If the process is non-Gaussian and H(z) is non-minimum phase, magnitude and phase can be correctly recovered *without* the true v[n]'s non-Gaussian distribution knowledge. This can be done estimating the ARMA parameters in an spectral domain of order higher than 2.

The output s.p. y[n] is said to be represented by an ARMA model when it satisfies the following difference equation:

$$y[n] + \sum_{k=1}^{P} a[k]y[n-k] = \sum_{k=0}^{Q} b[k]v[n-k] \quad (12)$$

In [2], Giannakis & Mendel show that using only finite samples of the autocorrelation $r_{yy}[m]$ and third-order cumulant $k_3^y[0,m,n_0]$ in a given slice n_0, it is possible to obtain the ARMA(P,Q) parameters a[k] and b[k]. In the frequency domain, $r_{yy}[m]$ corresponds to the conventional 1-D output spectrum that conveys information only on model's amplitude. The $k_3^y[0,m,n_0]$ statistics is equivalent to the 1½ -D output spectrum, that is, a projection of the bispectrum (2-D) that brings additional information to recover the original ARMA coefficients.

Adopting third-order cumulants, consider r=3 in Eq. (10):

$$k_3^v(0,m_1,m_2) = \begin{cases} \gamma_3^v & \text{, if } m_1 = m_2 = 0 \\ 0 & \text{, for other cases} \end{cases} \quad (13)$$

Using this result, rewriting Eq. (11) in time domain, making $m_1 = m_2 = m$ (diagonal slice), we have:

$$k[m] \equiv E\left\{y[n]y^2[n+m]\right\} \equiv k_3^y[0,m,m] = \gamma_3^v \sum_{i=0}^{\infty} h[i]h^2[i+m]$$

where h[i] is the system impulse response. (14

Eq. (14) has its analogous relation to the conventional autocorrelation-based parametric spectrum estimation as:

$$r_{yy}[m] = E\{y[n]y[n+m]\} = \sigma^2 \sum_{i=0}^{\infty} h[i]h[i+m] \quad (15)$$

where $\sigma^2 = r_{vv} = E\{v^2[n]\}$.

In the presence of i.i.d. Gaussian additive noise *n* (or even non-obliquous PDF when using k_3^y) with zero mean and variance σ_n^2, the observed (measured) data z[n] is described as:

$$z[i] = y[i] + n[i] \quad (16)$$

As we already seen, the cumulants for orders greater than 2 are not contaminated by this kind of observation noise:

$$k_z[m] = k_y[m] \quad (17\text{-a})$$

The same is not true for the autocorrelation zero-lag term:

$$r_z[m] = r_y[m] + \sigma_n^2 \delta[m] \quad (17\text{-b})$$

In fact there is a means of quantifying and removing the Gaussian observation noise influence from the parameter estimation algorithm, when the cumulant's approach is adopted.

III. Radar Sea Clutter - A Case Study

The advantages of cumulant-based spectral estimation over autocorrelation-based methods for non-Gaussian processes assumes an interesting feature when this process is Weibull distributed. This feature will be clarified in the following, as soon as we state the problem of Weibull radar sea clutter spectral modeling under additive Rayleigh observation noise.

The sea clutter data was collected from an air-traffic control radar located in Kanagawa - Japan, by Dr. Matsuo Sekine from Tokyo Institute of Technology [12]. The area under observation was within the 38.2° and 43.3° in azimuth and 2.0km to 5.5km in range. With the radar parameters it is possible to show that the radar resolution cells overlap only in the azimuth direction, 12.5 times for a punctual target [7], [13]. As we are interested in exploiting the correlation embedded in data for purposes of clutter spectral modeling, the preferred direction of processing is in azimuth. In fact the data is exploited in both directions, as will be seen later.

In a previous paper, the process was proved to be non-stationary [7]. Besides, using the Run test, it was shown to be locally stationary for less than 48 samples. It was also shown to obey Weibull distribution [12] which is defined as:

$$p_c(x) = \frac{c}{b} \cdot \left(\frac{x}{b}\right)^{c-1} \cdot e^{\left(-x/b\right)^c} \quad b > 0, c > 0, x > 0 \quad (18)$$

and zero for other cases, and c and b are the *shape* and *scale* parameters, respectively. Let's submit the available data x to the scale normalization process proposed for Weibull CFAR detectors [1], [9], [12], resulting in the normalized process z:

$$E\{\ln(x)\} = \ln(b) - \frac{\gamma}{c} \quad (19\text{-a})$$

$$z = e^{\ln(x) - E\{\ln(x)\}} \quad (19\text{-b})$$

$$p_c(z) = e^{-\gamma/c} \cdot c \cdot z^{c-1} \cdot e^{-\left(z^c e^{-\gamma}\right)} \quad c > 0, z > 0 \quad (19\text{-c})$$

where γ is the Euler constant. As can be seen, the z dependence on scale parameter b= $e^{\gamma/c}$ has been removed. The shape parameter, as its name indicates, defines the shape of the PDF curve, the exponential and Rayleigh distribution being special cases of Weibull distribution, when c=1 and c=2, respectively. The r*th* moment around origin μ_r for s.p. z is:

$$\mu_r(z) = e^{-\frac{r}{c}\gamma} \Gamma\left(\frac{r}{c} + 1\right) \quad (20)$$

For purposes of comparison between *spectral* and *ensemble* domain analysis, let's state now the ensemble domain results obtained by Sekine [12] for the specified clutter process. The x scale and shape parameters are extracted from data using the following relations, based on definition of X and Y:

$$X = \ln(x) \quad (21\text{-a})$$

$$Y = \ln\{-\ln[\,1 - \int_0^x p_c(x)dx\,]\} \quad (21\text{-b})$$

$$Y = cX - c \cdot \ln(b) \quad (21\text{-c})$$

Note that the parameter c can be seen as line $Y(X)$ derivative.

If we consider that sea surface does not change its state considerably during a small change in antenna's azimuth, we can say that all the samples in this azimuthal region are part of the same stochastic process: the sea clutter. Under this assumption, Fig. 1 shows a typical Y *versus* X curve for c shape parameter estimation, for 12 samples-azimuth and 200 samples-range totaling 2400 samples which we'll call a *"sector"* from now on.

If the data follow Weibull distribution, Fig. 1 must be a straight line, and its derivative is the c parameter. But there is a deviation from a straight line in the figure shown, specially for low values of X. This is due to presence of additive radar receiver noise (Johnson noise), whose distribution is Gaussian. Once this noise composes with the clutter signal after passing through an envelope detector, the measured signal is the combination of a Weibull s.p. (clutter) and a Rayleigh s.p. (noise). What differs one from another is just the c value, once Rayleigh distribution is Weibull distribution for c=2. By eliminating the samples with low amplitude (subject to greater noise influence) from processing, the clutter parameter shape for the case of Fig. 1 is shown to be c=1.2116. A straight line for c=3 is also shown for visual comparison. The same calculations are made for all the observed area, leading to sea clutter shape parameters ranging from 1.2 to 1.6.

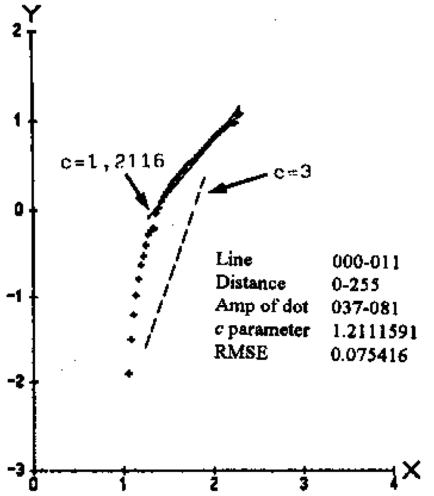

Fig. 1: Weibull radar clutter PDF parameter extraction from the ensemble domain analysis.

From Eq. (20), the corresponding values for μ_1 (mean) and μ_2 (covariance) can be calculated, which are essentially ensemble domain descriptors. Its counterpart equivalents in the spectral domain are the DC power and the total average AC power. The total normalized process power calculated in the ensemble domain is then $\mu_1+\mu_2$, which for the 21 sectors covered by the radar, ranges from 4.0 to 2.3, as shown as "ensemble domain" curve in Fig. 2-a.

IV. SPECTRAL VERSUS ENSEMBLE DOMAIN

An aspect of special interest is the way that correspondences are stated between the different processing domains, in order to make possible a precise numeric comparison. Each domain deals with its own describing parameters, and sometimes there is a need to compare equivalent parameters from different domains. As the sea clutter data under study obeys a Weibull distribution, a well defined relation between the shape and scale parameters of the PDF and the moments of the process allows a direct mapping between the ensemble and spectral domains: once that the second order moment corresponds to the autocorrelation, and its zero lag term corresponds to the total average power in the process, which is obtained also from the integrated power spectrum, there is a link between the parameters of the Weibull distribution (ensemble domain) and the power spectral density (spectral domain). The statistical description serves in this manner as a reference result, to which the results from the conventional second-order moment-based estimation, and from the third-order cumulant-based estimation have to be compared.

For conventional second-order spectral modeling, a previous study [6] involving almost all possibly applicable parametric ARMA modeling algorithms for these data indicated that the most appropriate choice should be a variation of Burg's method, originally described in [10]: the multisegmented Burg's method. It considers a number of short sequences obtaining a mean valued set of reflection coefficients. In this way The 21 *sectors* adopted for ensemble domain analysis (21 sets of 200 groups of 12-samples short sequences) can also be processed by the Burg's algorithm.

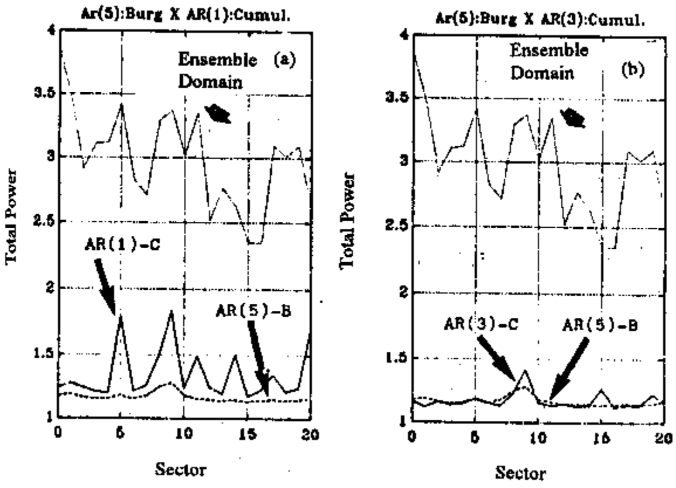

Fig. 2: Total process power per sector. Ensemble domain, second order spectral domain and third-order bispectrum domain analysis results comparison.

It can be showed [6], [8], that an originally AR(P) process contaminated by additive white noise, becomes an ARMA (P,P) process. In the ensemble domain analysis, the receiver noise influence was reduced by discarding low values in Fig. 1 in the estimation of the shape parameter. In the case of second-order AR modeling, this influence reduction is not so simple. It was observed from the conventional analysis, that uncontaminated sea clutter should be well described as an AR(3) process, but because of noise influence, the final contaminated clutter should be modeled by an AR(5) or an ARMA(3,3) process. Fig. 3-a shows the contaminated clutter autocorrelation sequence for orders 3, 5 and 9, highlighting the strong correlation for 12 successive resolution areas. Fig. 3-b shows the results of two equivalent order estimation criterion, AIC and FPE [8]. The ARMA(3,3) was not adopted due to excessive increase in algorithm complexity, once the AR(5) modeling was considered a satisfactory result. Fig. 3-c brings the residue autocorrelation. Fig. 3 serves to highlight the amount of negative influence of additive noise if we compare changes from AR(3) to AR(5) models.

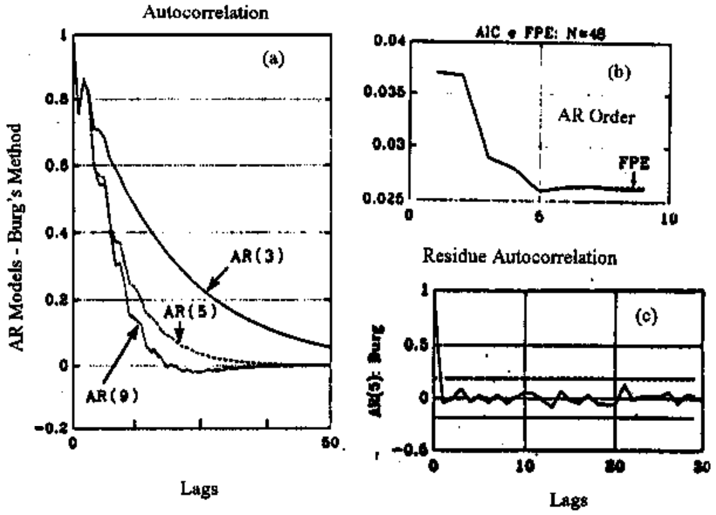

Fig. 3: Observation noise influence over conventional AR modeling: (a) Clutter autocorrelation sequence; (b) AIC and FPE order estimation; (c) Residue autocorrelation sequence.

The normalized $z[k]$ sea clutter s.p. mean spectral shape, resulting from second-order AR(5) modeling, can be viewed as the curve labeled "Burg" in Fig. 4. As we know, the total area under power spectral density (PSD) curve is the total average normalized power of the process (or the zero lag autocorrelation element). If we integrate the PSD obtained for the same 21 sectors previously adopted for ensemble analysis, we obtain the curve labeled AR(5)-B in Fig. 2. We can see from this curve that its variation is much less than the ensemble domain curve, from sector to sector, and ranges from 1.2 to 1.27. From Eq. 20, it is known that these zero lag autocorrelation values correspond to shape parameters ranging from c=3.9 to c=3.5, respectively. For practical purposes, shape parameters around c=3.5 turns the Weibull PDF into an almost Gaussian PDF. It fails to increase model order aiming to obtain shape parameters near the 1.2 to 1.6 values from ensemble domain analysis.

Once the additive noise is Rayleigh distributed, its shape parameter is equal to 2. So, it is "closer" to the 3.5 value that characterizes the Gaussian PDF than the clutter itself, whose shape parameters assume values between 1.2 and 1.6. So, if we adopt third-order cumulant-based modeling, though we are not able to completely eliminate noise influence (once it is not Gaussian), this influence is strongly minimized, as can be seen from the curve labeled AR(1)-C in Fig. 2-a, and for AR(3)-C curve in Fig.2-b, for first-order and third-order AR modeling, respectively. In Fig. 2-a, the variations for AR(1)-C curve are greater than for AR(5)-B, and are near the ensemble domain curve. In this case, the corresponding shape parameters range from c=3,5 to c=1,56.

The process in this case was successfully modeled by a first order model, the third-order model being showed for purposes of comparison. Fig. 5-a and Fig. 5-b shows the cumulant-based model residue autocorrelation and residue cumulants, respectively. Fig. 5-c shows a comparison of residue variance versus model order for both cases, that is, Burg's model and Cumulant-based model. It is clearly shown that an AR(1) model obtained from cumulant's approach is superior to the AR(5) model obtained from second-order statistics. So we achieved a better quality model with lesser order. The price paid for this was the increase in algorithm complexity. An ARMA(3,3) model was also tested, but the high variance of filter poles and zeros showed it was not a good model for this process. The cumulant-based PSD shape is shown in Fig. 4.

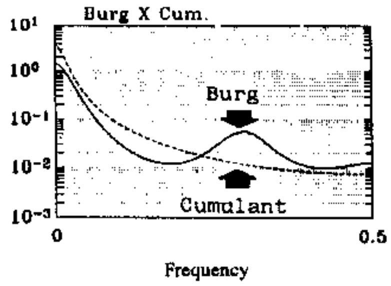

Fig. 4: Sea clutter mean power spectral density. Burg's AR(5) modeling *versus* Cumulant's AR(1) modeling.

V. CONCLUSIONS

We have presented a well defined correspondence between spectral and ensemble domains, for the case of a Weibull distributed process. After, we used this correspondence to compare the cumulant-based AR spectral modeling approach to the conventional second-order Burg's maximum entropy approach, using the ensemble domain results as a base. The result was a decrease in AR model order from 5 to 1, with the results from spectral domain becoming closer to the results in the ensemble domain. The cumulant-based model exhibited less residue variance and was less influenced by Rayleigh distributed receiver noise.

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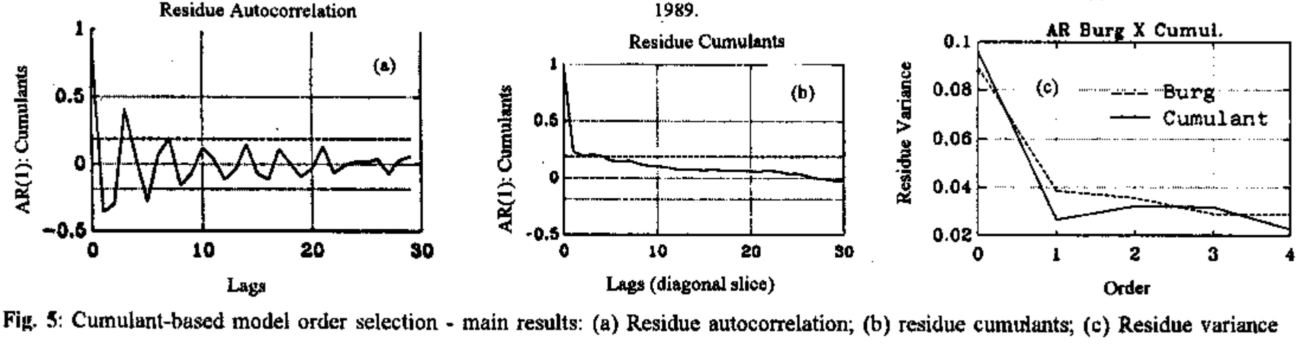

Fig. 5: Cumulant-based model order selection - main results: (a) Residue autocorrelation; (b) residue cumulants; (c) Residue variance