

Hodgkin-Huxley Model

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Introduction

The **Hodgkin-Huxley Model** is a set of non-linear ordinary differential equations that approximate the electrical characteristics of excitable cells such as neurons

Biophysical Analogue

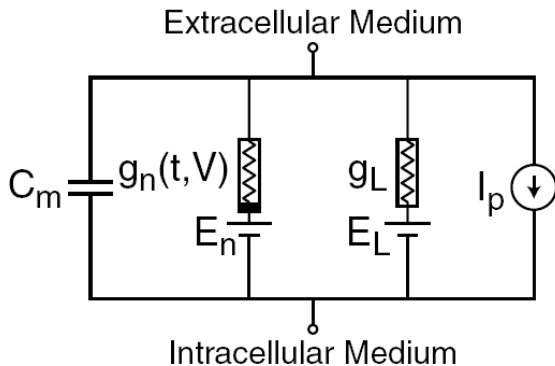


Figure: Hodgkin-Huxley

Parameters

- ▶ The lipid bilayer is represented as a capacitance (C_m).
- ▶ Voltage-gated ion channels are represented by nonlinear electrical conductance (g_n , where n is the specific ion channel).
- ▶ Leak channels¹ are represented by linear conductances (g_L).
- ▶ The electrochemical gradients driving the flow of ions are represented by batteries (E_i and E_L), the values of which are determined from the Nernst potential² of the ionic species of interest.
- ▶ Ion pumps are represented by current sources (I_p).

¹Leakage channels, also called Resting channels, are ion channels in the plasma membrane of a cell that remain open at all times.

²The potential level across the cell membrane that exactly opposes net diffusion of a particular ion through the membrane is called the Nernst potential for that ion.

Hodgkin-Huxley Equation

$$\frac{\partial V_m}{\partial t} = \frac{1}{C_m} \sum_i I_i \quad (1)$$

- ▶ V_m : the potential across the membrane.
- ▶ I_i : individual ionic currents of the model.

Current Flow

The current flowing through the ion channels is a function of V_m (the membrane potential) and t (time).

$$I_i(V_m, t) = g_i(V_m - E_i) \quad (2)$$

In voltage-gated ion channels, the channel conductance g_i is a function of both time and voltage ($g_n(t, V)$), while in leak channels g_i is a constant (gL).

Voltage-Gated Ion Channels

Conductances for voltage-gated channels are expressed as:

$$g_n(V_m, t) = \bar{g}_n \phi^\alpha \chi^\beta \quad (3)$$

where ϕ and χ are gating variables for activation and inactivation, respectively, representing the fraction of the maximum conductance available at any given time and voltage.

$$\frac{\partial \phi}{\partial t}(V_m, t) = \frac{1}{\tau_\phi}(\phi_\infty - \phi) \quad (4)$$

$$\frac{\partial \chi}{\partial t}(V_m, t) = \frac{1}{\tau_\chi}(\chi_\infty - \chi) \quad (5)$$

τ_ϕ and τ_χ are time constants; ϕ_∞ and χ_∞ are the steady state values for activation and inactivation, respectively.

Nobel Prize



The Nobel Prize in Physiology or Medicine 1963

"For their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the peripheral and central portions of the nerve cell membrane"



Sir John Carew Eccles

🏆 1/3 of the prize

Australia

Australian National
University
Canberra, Australia

b. 1903
d. 1997



Alan Lloyd Hodgkin

🏆 1/3 of the prize

United Kingdom

University of
Cambridge
Cambridge, United
Kingdom

b. 1914
d. 1998



Andrew Fielding Huxley

🏆 1/3 of the prize

United Kingdom

London University
London, United
Kingdom

b. 1917

Figure: Nobel Prize



How to Model?

How do we model a real neuron?

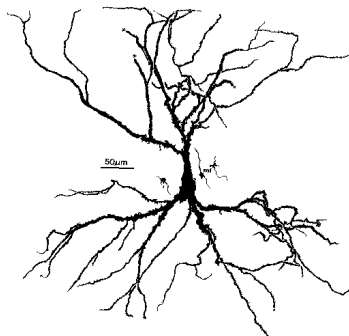


Figure: Real Cell

Compartmental Model

We model it piece by piece.

The usual approach is to model this with a lumped parameter model in which we divide the neuron into a finite number of compartments containing resistances, capacitances and batteries to represent ionic equilibrium potentials.

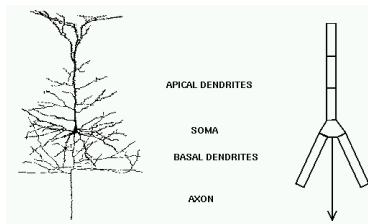


Figure: Piece by piece

Purkinje Cell Example

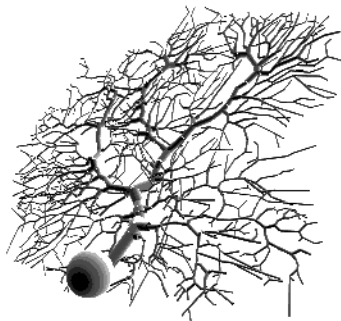


Figure: Purkinje Cell Model on GENESIS

Compartments

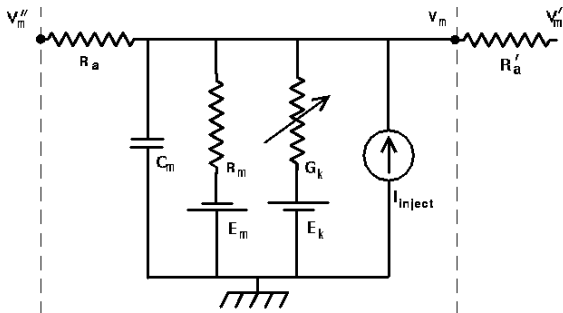


Figure: Compartment equivalent circuit



Hodgkin, A. and Huxley, A. (1952).

A quantitative description of membrane current and its application to conduction and excitation in nerve.

Journal of Physiology, 117:500–544.